Multi-lag Processing to Improve Polarimetric Radar Data Quality

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Outline

• Existing estimators
• Multi-lag estimators
• Test with simulations
• Test with OU’ data
• Summary
Moment data are obtained from correlation estimates

• Correlation functions are estimated from time-series data (Doviak & Zrnic 1993)

\[
\tilde{C}_{h,v}(nT_s) = \frac{1}{M - n} \sum_{m=1}^{M} V^*_{h,v}(mT_s) V_{h,v}((m + n)T_s)
\]

\[
\tilde{C}_{hv}(nT_s) = \frac{1}{M - n} \sum_{m=1}^{M-n} V^*_h(mT_s) V_v((m + n)T_s)
\]

• Auto- and Cross-correlation function

\[
C_{h,v}(nT_s) = S_{h,v} \rho(nT_s) \exp\left(-\frac{j\pi n\bar{v}}{v_N}\right) + \delta_{h,v} \delta_n
\]

\[
C_{hv}(nT_s) = \sqrt{S_h S_v} \rho_{hv} \rho(nT_s) \exp\left(-\frac{j\pi n\bar{v}}{v_N}\right) + j\phi_{dp}
\]
Conventional estimators

• Widely accepted and used (Doviak&Zrnic 1993)

\[ \hat{S}_{h,v} = \bar{C}_{h,v}(0) - \hat{N}_{h,v} \]

\[ \hat{Z}_{DR} = 10 \cdot \log_{10} \left( \frac{\hat{S}_h}{\hat{S}_v} \right) \]

\[ \hat{\rho}_{hv} = \frac{|\bar{C}_{hv}(0)|}{(\hat{S}_h \cdot \hat{S}_v)^{1/2}} \]

\[ \hat{S}_{h,v} = \frac{\lambda}{2 \sqrt{2\pi T_s}} \sqrt{\ln(|\hat{S}_{h,v}|) - \ln(|\bar{C}_{h,v}(T_s)|)} \]

– Variable noise, difficult to estimate
– Error in Z_{DR}
– Reduced \rho_{hv}
Lag-one estimator

- Melikov et al. (2004/2006) developed

\[
\hat{S}_{h,v}^{(1)} = |\hat{\Theta}_{h,v}(T_s)|
\]

\[
\hat{Z}_{DR}^{(1)} = 10 \cdot \log_{10} \left( \frac{|\hat{\Theta}_h(T_s)|}{|\hat{\Theta}_v(T_s)|} \right)
\]

\[
\hat{\rho}_{hv}^{(1)} = \frac{|\hat{\Theta}_{hv}(-T_s)| + |\hat{\Theta}_{hv}(T_s)|}{2 \cdot [\hat{\Theta}_h(T_s) \cdot \hat{\Theta}_v(T_s)]^{1/2}}
\]

- Biased estimates for power, large errors in $Z_{DR}$ and $\rho_{hv}$
Multi-lag processing
(Cao et al. 2010, Lei et al. 2011, Zhang et al. 2004)

• Existing moment estimators depend mainly on lag-zero and lag-one correlation estimates that are contaminated by noise and estimation error

• The multi-lag estimator, excluding lag-zero auto-correlation and using more available lags, is introduced

• Because lag-zero of auto-correlation estimates is excluded, it is expected to data quality can be improved at low SNR.
Multi-lag processing

- Gaussian fit to multi-lag correlation estimates (depending on $\sigma_v$ and SNR).
Fitting procedure

• Zero-mean Gaussian correlation function with two parameters

\[ y_n = \ln(|C_{h,v}(nT_s)|) = an^2T_s^2 + b \]

\[ \chi(\ddot{\phi},\ddot{\theta}) = \sum_{n=1}^{N} (n^2 \ddot{\phi}T_s^2 + \ddot{\theta} - \ddot{y}_n)^2 \]

\[
\begin{cases}
\frac{\partial \chi(\ddot{\phi},\ddot{\theta})}{\partial \ddot{\phi}} = 0 \\
\frac{\partial \chi(\ddot{\phi},\ddot{\theta})}{\partial \ddot{\theta}} = 0
\end{cases}
\]

\[
\ddot{\phi} = \frac{30 \sum_{n=1}^{N} [6n^2 - (N+1)(2N+1)] \ddot{y}_n}{T_s^2 N(N-1)(N+1)(2N+1)(8N+11)}
\]

\[
\ddot{\theta} = \frac{6 \sum_{n=1}^{N} (3N^2 + 3N - 1 - 5n^2) \ddot{y}_n}{N(N-1)(8N+11)}
\]
Bias of spectrum width estimates
Standard deviation of spectrum width estimates
Bias of differential reflectivity estimates
Standard deviation of differential reflectivity estimates
Test with simulated data

- NWP model (ARPS) by predicts wind field and microphysical variables: number concentration and water mixing ratios for cloud water & ice, rain, snow and graupel
- True radar variables are calculated.
- Time-series data generated
Simulation based on ARPS output

• Time series generated from NWP model output
  Signal amplitudes and phases determined from ARPS (Xue et al. 2000): mixing ratio for rain, snow and hail, wind, and turbulence.


Zhang, G, 2008: METR/ECE 6613 - Weather Radar Polarimetry, lecture notes,
Comparison of Reflectivity

$\text{SNR} = 5\text{dB}$
Comparison of $Z_{DR}$
Comparison of $\rho_{hv}$
Adaptive fitting

• Adaptively determine the usable lags.
  – Error check (consistence, fitting)
• Use adaptive weighting for fitting.
  – Minimize the cost function

\[ \chi = \sum_{n=1}^{N} \left| \hat{C}_n - |C(nT_s)| \right|^2 |C(nT_s)|, \]

\[ |C(nT_s)| = C(0) \exp \left( -\frac{(nT_s)^2}{2\tau_c^2} \right). \]
Comparison of spectrum width

- The wide spectra caused by noise reduced
Comparison of correlation coefficient

- Correlation coefficient increased for weak echoes
Summary and discussions

• There is deficiency with existing moment estimator
• Multi-lag estimator is introduced to improve PRD quality
• The improvement has been demonstrated using simulated and OU’ data
  – Significant improvements for power, $\sigma_v$ and $\rho_{hv}$ estimation.
  – Marginal Improvements for $Z_{DR}$, $v_r$, and $\Phi_{DP}$
• It is expected more improvement with S-band PRD because of the longer correlation time.
• Look forward to testing/implementing with KOUN data